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6 How To Complete The Square

Method 1: Follow A Technique

Example with a 1 in front of x^2

$$x^2 - 8x - 1$$

Step 1: Halve the number in front of x and put it in squared bracket that look likes $(x+?)^2$ or $(x-?)^2$.

$$\frac{-8}{2} = -4$$

$$(x-4)^2$$

Step 2: Copy the constant at the end

$$(x-4)^2 - 1$$

Step 3: undo (subtract) the 4^2 above

This means, $(x-4)^2 - 1$ in step 2 becomes

$$(x-4)^2 - 1 - 4^2$$

Step 4: Simplify

$$(x-4)^2 - 17$$

You might be wondering. Why did we do -4^2 in step 3? Or even why these steps even work?
If we expand $(x-4)^2 - 1$ in step 2 we get $x^2 - 8x + 16 - 1$
BUT, we had $x^2 - 8x - 1$ in the original question.
So the extra term appearing is $+16$ which is 4^2. This is why we undo it/need to get rid of it hence the -4^2

Example WITHOUT a 1 in front of x^2

$$2x^2 - 5x - 3$$

We need to do an extra step first which is to FACTOR OUT whatever number is in front of x^2 and then we complete the square after. There are 2 ways to factor out the number first of all. We can either

Way 1: Factorise the 2 out of the **first 2 terms** ONLY

Step 1: We need to factorise out the 2 first, from the first 2 terms only. This just divides all terms by 2

$$2\left(x^2 - \frac{5}{2}x\right) - 3$$

Step 2: Now complete the square on what is inside the bracket. Some students get confused since there aren't 3 terms inside brackets like when usually completing the square. The third term that we usually is just 0 now hence nothing to worry about.

$$= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] - 3$$

$$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] - 3$$

Step 3: Multiply the 2 back in

$$= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} - 3$$

Step 3: Simplify

$$= 2\left(x - \frac{5}{4}\right)^2 - \frac{49}{8}$$

Way 2: Factorise the 2 out of **ALL 3 terms**

Step 1: We need to factorise out the 2 first from ALL terms. This just divides the first two terms by 2

$$2\left(x^2 - \frac{5}{2}x - \frac{3}{2}\right)$$

Step 2: Now complete the square on inside the bracket as usual.

$$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{3}{2} - \left(\frac{5}{4}\right)^2\right]$$

Step 3: Simplify

$$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{3}{2} - \frac{25}{16}\right]$$

$$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{49}{16}\right]$$

Step 4: Multiply the 2 back in

$$= 2\left(x - \frac{5}{4}\right)^2 - \frac{49}{8}$$

Method 2: Memorise a template

Form 1: Quadratics $x^2 + bx + c$ become: $\left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$

$$x^2 - 8x - 1$$

Here $b = -8, c = -1$

becomes

$$\left(x + \frac{-8}{2}\right)^2 + -1 - \left(\frac{-8}{2}\right)^2$$

Simplify

$$(x-4)^2 - 1 - (-4)^2$$

$$(x-4)^2 - 1 - 16$$

$$= (x-4)^2 - 17$$

Form 2: Quadratics $ax^2 + bx + c$ become $a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right]$

$$2x^2 - 5x - 3$$

Here $a = 2,\ b = -5, c = -3$

becomes

$$2\left[\left(x + \frac{-5}{2(2)}\right)^2 + \frac{-3}{2} - \left(\frac{-5}{2(2)}\right)^2\right]$$

Simplify

$$= 2\left[\left(x - \frac{5}{4}\right)^2 - \frac{3}{2} - \frac{25}{16}\right]$$

$$= 2\left(x - \frac{5}{4}\right)^2 - \frac{49}{8}$$

Method 3: Expand answer form and compare coefficients

$$x^2 - 8x - 1$$

We know that our answer form will look like $(x+p)^2 + q$
Expanding this gives $x^2 + 2px + p^2 + q$

So $x^2 - 8x - 1$ is identical to ($\equiv$) $x^2 + 2px + p^2 + q$

Let's colour code for ease of explanation:

$$x^2 - 8x - 1 \equiv x^2 + 2px + p^2 + q$$

By comparing coefficients of the x^2, x and constant terms we get

$$2p = -8$$

$$p^2 + q = -1$$

Solving simultaneously via substitution gives,

$$p = -4, q = -17$$

So $(x+p)^2 + q$ becomes $(x-4)^2 - 17$

$$2x^2 - 5x - 3$$

Our answer form will look like $a(x+p)^2 + q$
Expanding this gives $ax^2 + 2apx + ap^2 + q$

So $2x^2 - 5x - 3$ is identical to ($\equiv$) $ax^2 + 2apx + ap^2 + q$

Let's colour code for ease of explanation:

$$2x^2 - 5x - 3 \equiv ax^2 + 2apx + ap^2 + q$$

By comparing coefficients of the x^2, x and constant terms we get

$$a = 2$$

$$2ap = -5$$

$$ap^2 + q = -3$$

Solving simultaneously via substitution gives, $a = 2,\ p = -\frac{5}{4},\ q = -\frac{49}{8}$

so $a(x+p)^2 + q$ becomes $2\left(x - \frac{5}{4}\right)^2 - \frac{49}{8}$

Examples where you need to re-arrange first

$$5 + 2x - x^2$$

Re-write as

$$-x^2 + 2x + 5$$

We now factorise out the -1 (i.e. divide everything by 2)

$$-1(x^2 - 2x - 5)$$

Now complete the square on inside the bracket as usual

$$-1[(x-1)^2 - 5 - 1]$$

$$-1[(x-1)^2 - 6]$$

Multiply the -1 back in

$$-1(x-1)^2 + 6$$

$$6 - (x-1)^2$$

$$1.8 + 0.4d - 0.002d^2$$

Re-write as

$$-0.002d^2 + 0.4d + 1.8$$

We now factorise out the -0.002 (i.e. divide everything -0.002)

$$-0.002(d^2 - 200d - 900)$$

Now complete the square on inside the bracket as usual

$$-0.002[(d-100)^2 - 900 - 100^2)]$$

$$-0.002((d-100)^2 - 10900)$$

Multiply the -0.002 back in

$$-0.002(d-100)^2 + 21.8$$

$$21.8 - 0.002(d-100)^2$$